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Market-implied risk-neutral probabilities, actual probabilities, credit risk and news

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Abstract Motivated by the credit crisis, this paper investigates links between risk-neutral probabilities of default implied by markets (e.g. from yield spreads) and their actual counterparts (e.g. from ratings). It discusses differences between the two and clarifies underlying economic intuition using simple representations of credit risk pricing. Observed large differences across bonds in the ratio of the two probabilities are shown to imply that apparently safer securities can be more sensitive to news.

Introduction

The impact of the recent credit crisis on markets, banks, and real economic performance underscores the importance of the valuation of credit risk and the role of default probabilities. Central to this issue are the uses of both *market-implied Risk-Neutral Probabilities of Default* (hereafter, RNPDs) and *Actual Probabilities of Default* (hereafter, APDs). This paper investigates links between the two sets of probabilities and clarifies underlying economic intuition using simple representations of credit risk pricing. It explains why bonds with lower actual default probabilities may have differentially stronger reactions to news.

By definition, actual probabilities of events represent measures of their likely occurrence in the real-world. Assessments of creditworthiness by rating agencies, for example, have historically been based on APDs estimated using firm-level information, other data such as past default experience, and analyst expertise. Thus, a credit rating of AAA by Standard and Poor's is typically meant to reflect lower estimated odds of actual future default than those corresponding to a BBB rating.[1]

A different approach is to try and extract investors' expectations from asset market prices. It has the benefits of relying on real-time data that aggregates the potentially disparate information and opinions of market participants besides being forward-looking in nature.[2] However, just as

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[1] Standard and Poor's and CRISIL make explicit reference to their ratings reflecting real-world default frequencies while Moody's makes particular mention of associated expected losses.

[2] Such uses of market prices by traders, central banks, and firms goes back many decades. Examples range from the estimation of expectations of future interest rates from the yield curve to those of dividend growth rates from stock prices.

the estimates of a ratings analyst depend on his 'views' of the future, the measurement of market-implied probabilities must also necessarily use a 'model' or assumptions.[3] The reason is that prices depend on more than just the odds of occurrence of future events (and corresponding cash flows) – they also involve opportunity costs. So-called market-implied risk-neutral probabilities in general, including those pertaining to default, i.e. RNPDs, are obtained by utilising one such assumption, viz. that investors are risk-neutral and desire no risk premia.

The assumption of risk-neutrality is obviously counterfactual because typical investors are risk-averse and desire compensation for bearing risk. Accordingly, the traditional practice to pricing a credit-sensitive security sets its yield spread (relative to the riskfree interest rate) to reflect required returns that are commensurate with risk premia on similar assets in addition to paying investors actuarially fair compensation for any losses from default. On the other hand, under the assumption of risk-neutrality, a spread is purely the expectation of the possible loss – computed with risk-neutral probabilities. This both serves as a definition of risk-neutral probabilities and a means of constructing market-implied RNPDs from spreads. Consistency between these two ways of expressing spreads implies that an RNPD is linked to its APD counterpart with an adjustment made for risk premia.

Thus, unlike APDs, which are in principle unbiased predictors of future empirical or realised default frequencies, RNPDs are by design almost surely bound to be *wrong* as measures of real-world odds. Instead, the RNPD of a bond can be shown to be the market-implied value of the insurance premium on a policy that insures a bond against default. Risk-neutral probabilities are remarkably useful in valuation as first articulated in the pricing of stock options and later with credit derivatives precisely because they represent implied costs of insuring against various events. But how useful RNPDs may be as pure indicators of likely default does not appear to be entirely well-understood as evidenced by a variety of interpretations among credit market participants and observers that one encounters in this context. Examples of news reports from the financial press point to a mixed recognition of the fact that the RNPD of an issuer is not necessarily a signal of only its likely default. We show that, according to the preceding link, an RNPD is affine in its APD with weights in this linear transformation involving the risk premium. Thus, the common belief that changes in one probability are mirrored by similar changes in the other may stem from a tendency to ignore the endogenous, time-varying nature of these weights or risk premia.

Our paper sheds light on what Hull, Pedrescu & White (2005) term the *Credit Risk Puzzle*. They show that RNPDs averaged across ratings classes were 5.7% per year and exceeded the average annual APDs of 3.7% in their sample, and argue that this is consistent with risk-averse investors requiring risk premia.[4] A further striking aspect to the Hull, Predescu, and White (2005) data is that whereas the ratio of the (preceding) average RNPD to the average APD is about 1.5, the average of the ratio of the two probabilities is about 7. In other words, the relationship between the two probabilities is significantly non-linear in the cross-section of ratings. For instance, Aaa-rated bonds' RNPDs are about 17 times as large as their actual default probabilities whereas the comparable ratio for 'less safe' Ba-rated bonds is only about 2. However, previous work has for the most part not commented on the implications of this cross-sectional regularity which clearly casts doubt on a narrow, linear interpretation of the APD-RNPD relationship.[5] If the excess of an RNPD over its APD counterpart is due to a risk premium, what explains the tendency for the *highest* rated bonds to have the largest RNPD/APD ratios? Put differently, while an Aaa-rated bond is only 1/60th as likely to default compared to a Ba-rated one, market prices imply that insurance against default of the Aaa is only about 1/8th as cheap. A priori, one might expect the role of risk premia (or costs of insurance) to be least significant for these bonds if risks were determined solely by APDs. This non-linear RNPD/APD relationship also has implications for the impact of news, i.e. changes or innovations in APDs. We show that news may have differentially greater impact on apparently safer bonds given the preceding tendency for higher-rated debt to sell at larger RNPD/APD ratios.

While the general fact that actual and risk-neutral probabilities differ is well-known in the academic theory of asset pricing (see, for e.g., Dybvig and Ross (1987), there is a paucity of analyses and discussions of the determinants of their mutual links and the resulting implications for credit risk. Apart from the work cited earlier, there are a few published studies of APD-RNPD links including Almeida and Philippon (2007), Bohn (2000), Coval, Jurek, and Stafford (2008), Delianedes and Geske (1998), Hund (2003), Kealhofer (2003), and Berg (2010). However, these papers do not discuss all issues related to differences between the two such as biases in the cross-section or time-series. It is also important to note that all these papers (except Coval et al) obtain their results working largely within a Merton (1974)-type framework. This is significant because the Black-Scholes-Merton framework has heavily influenced much academic work and practice.[6] Crucially, as

[3] Inferences are drawn by fitting a pricing model (e.g. the Black and Scholes (1973) option pricing model when constructing implied volatilities). One determines what must be true for the numerical value of a particular probability (or related moment such as a standard deviation) which when input into the model produces an estimate of an asset's value that matches its observed market price.

[4] Their data consists of yield spreads on 7-year corporate debt from 1996 to 2004 and historical default frequencies from Moody's over 1970—2003.

[5] Hull et al. (2005) and Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) remark that the ratio is higher for better quality firms.

[6] The Merton (1974) conceptualisation of debt pricing is at the core of some businesses who produce and market default probabilities, e.g. Moody's KMV. This approach is also considered to be a viable practice in meeting bank capital regulations. Nevertheless, Merton (1974) is by no means exhaustive and others have extended it to include more realistic features such as possible default before maturity, incomplete information about a firm's liabilities, etc. These extensions are primarily devoted to pricing and do not present systematic evaluations of the links between APDs and RNPDs.

Merton (1974, page 459) himself points out, his framework does not directly take into account factors such as the correlations of an issuer's assets with other assets in the economy which would influence the betas of a firm and its debt. Hence, we submit, it is of interest to examine whether other frameworks can shed light on understanding differences between RNPDs and APDs.

Hund (2003) explicitly examines the volatility properties of RNPDs in a Merton model but does not discuss any differences between RNPDs and APDs. Berg (2010) shows that, in a Merton framework, credit risk premia are a significant part of model spreads and their importance relative to expected losses increases as credit quality improves; i.e. RNPD-to-APD ratios increase as actual default probabilities fall. However, Berg does not offer any economic reason or intuition for why his result obtains other than suggesting that it has to do with the betas of bonds. This leaves unanswered questions of whether the result is particular to the general Merton framework and its ingredients such as the assumption of continuous trading.

Our approach is different from the papers cited above in that it focuses on the roles of investor risk aversion and multiple risky investment alternatives in the pricing of credit risk within a state-preference setting.[7] We argue that risk premia due to systematic risk factors explain the cross-sectional differences in RNPDs and APDs. Furthermore, the empirical result on the RNPD-to-APD ratio from Hull et al. (2005) is exactly what one would expect to hold for the average asset based on economic intuition relating to systematic risk. Thus, rationalisation of this observed regularity is robust to alternative model specifications vis-à-vis Merton (1974). However, unlike Berg (2010) where all issuers are restricted to have the same correlation with an investor's pricing benchmark (such as the market portfolio in the CAPM), a sub-investment grade issue can have a higher risk premium per unit expected loss than an investment grade bond; i.e. the RNPD-APD ratio need not be decreasing in the APD for all securities. Our approach is quite similar to that of Coval et al. (2008) but we differ in our objective of explaining the RNPD-APD relationship for bonds whereas they focus on structured products. (A further discussion of their paper is contained in the section '*RNPD/APD ratios and systematic risks in the cross-section of bonds*') We also show that bonds with high ratios are shown to be most sensitive to news. Finally, we believe that our paper illustrates the need for nuanced interpretation of market-implied probabilities given the impossibility of model-free constructions. This point is important in a policy context given recent calls to supplement (or even do away with) ratings issued by agencies in favour of 'market-implied ratings' (see Flannery, Houston, and Partnoy (2010).

[7] This framework, due to Arrow (1964) and Debreu (1959), underpins virtually all subsequent results on risk and return in financial economics. In using it, and by eschewing the complicated mathematical apparatus of modern derivatives valuation, we are able to provide refinements to intuition about qualitative effects but, unlike the typical structural and reduced form models, have to stop short of providing quantitative detail to the degree needed for pricing or trading.

The next section sets out the basic issues and shows the naïve affine relationship between RNPDs and APDs. An investor's optimisation problem is considered in the third section so as to draw explicit links to systematic risk. The fourth section offers further discussion, after which we conclude.

A basic framework and the issues

Consider assets that payoff at a future date T and are valued at an initial date 0. They include a default-free bond as well as risky bonds, all of which mature at date T with promised maturity payments of 1 each (in dollars, for ease). If a risky bond j defaults it pays $R_j < 1$, an assumed known recovery per unit promised payment. Uncertainty about the payoffs of all assets is captured by supposing there are alternative states of the world, $\{s\}$, of which one will come to prevail at date T.

Investors' beliefs about future uncertainty are represented by a probability measure P that assigns positive probabilities $\{p(s)\}$ to the states $\{s\}$. These are termed 'actual' probabilities since they refer to measures of the physical frequencies of occurrence of the states. The set of states in which an individual bond j defaults is denoted d_j with corresponding actual default probability $p_{d,j}$; the complementary set in which no-default occurs is n_j. A simple illustrative numerical example now follows.

Example 1. Let $T = 1$ year, and suppose there are only 3 states with probabilities as shown. There is 1 riskfree and 2 risky zero coupon bonds with payoffs across states as shown. Both risky bonds j pay $R_j = 0.6$ when in default.

States at date T	$s = 1$	$s = 2$	$s = 3$
Actual probabilities	$p(1) = 0.7$	$p(2) = 0.2$	$p(3) = 0.1$
	Future payoffs		
Riskfree bond	1	1	1
Risky bond A	1	1	0.6
Risky bond B	1	0.6	1

Clearly, bond A and B have APDs of $p_{d,a} = p(3) = 0.1$ and $p_{d,B} = p(2) = 0.2$ respectively.

Note that in Example 1 each bond satisfies a binomial-type representation familiar from option pricing models. However, the above also represents the risky bonds jointly; it involves an assumption on the correlation of their defaults.

Market-implied risk-neutral probabilities and no-arbitrage

The price $B_{0,j}$ of risky bond j must equal its expected future payoff discounted at a risk-adjusted rate given by the riskfree rate r_f plus a risk premium λ_j:

$$B_{0,j} = \frac{(1 - p_{d,j}) + p_{d,j}R_j}{1 + r_f + \lambda_j} \tag{1}$$

Using the yield $y_j \equiv 1/B_{0,j} - 1$ and the possible loss $1-R_j$ from default, (1) is equivalent to a yield spread for the bond of

$$y - r_f = \lambda + \frac{p_d(1-R)}{B_0}. \tag{2}$$

i.e. a bond holder is compensated with a risk premium in addition to the actual expected loss (rate) on his investment of $B_{0,j}$. Given an observed bond price (which necessarily satisfies (1) above), one can determine the value of $q_{d,j}$ such that $B_{0,j}$ also satisfies a version of (1) with a zero risk premium:

$$B_{0,j} = \frac{(1-q_{d,j}) + q_{d,j}R_j}{1+r_f}. \tag{3}$$

This is equivalent to expressing the bond's yield spread purely as an expected loss rate

$$y - r_f = \frac{q_d(1-R)}{B_0}, \tag{4}$$

computed with a new probability $q_{d,j}$. If $q_{dj} = [1 - B_{0,j}(1+r_f)]/(1-R_j)$, the solution, is positive and less than 1 then, as per our discussion in the Introduction, it is the RNPD for bond j. For this to be true, the risky bond's market price must be less (more) than a default-free counterpart with maturity payment of 1 (respectively, of R_j), abstracting from frictions such as short sale constraints, brokerage fees, taxes, etc. Otherwise, one could earn riskless arbitrage profits and the market-implied probabilities would take absurd values. Hence, in order for implied probabilities to have any merit, we assume all assets trade with no-arbitrage opportunities possible as in the following illustrative example.

Example 2. Consider Example 1 and suppose we are given observed current (date 0) prices for bonds A and B of 0.8 and 0.727 (or 8/11), respectively, with the default-free bond selling at a riskfree rate of 10%. The RNPDs implied by these prices as per (3) are $q_{d,A} = 0.3$ and $q_{d,B} = 0.5$.[8]

The 'credit spread puzzle'

Hull, Pedrescu & White (2005) present data that they term the *Credit Spread Puzzle*. Using default rates published by Moody's for the period 1970—2003, they estimate actual default intensities for 7-year corporate debt. These annualised default intensities 'h' are converted to our annual APDs as per $p_d = 1 - e^{-h}$. Using market data (Merrill Lynch bond indexes for 1969—2004) on corporate debt of 6.5—8.9 year maturity for the period 1996—2004, and an assumed recovery rate $R = 0.4$, Hull et al. (2005) also infer risk-neutral intensities from observed yield spreads $y - r_f$ which are converted to our RNPDs similar to our APDs.[9] Table 1 reports these APDs and RNPDs adapted from the Hull data. Also shown in the table (in basis points) are the actual expected losses $p_{d,j}(1-R)$ and the risk premium $\lambda \approx (q_d - p_d)(1-R)$ that come from the approximate versions of (2) and (4).

Hull et al. (2005) refers to the magnitude of the differences between the RNPDs and the APDs as the *Credit Spread Puzzle*. They, and others, focus on the differences in levels *averaged* across ratings classes and point to positive risk premia as explaining RNPDs greater than APDs (and thus ratios above 1). Also note that within each rating class, the Hull et al. (2005) data pertains to bond indexes or averages across bonds. However, as is clear from Table 1, the difference in the levels *and* the ratio of the two probabilities differ dramatically across bond ratings, and the relationship between the two is highly non-linear.[10] This prompts the question: Why are investment grade bond indexes so different from speculative grade bonds indexes in this respect?

Now consider another example that builds on, and yet differs from, the earlier Example 2. The latter featured two bonds with RNPD-to-APD behaviour qualitatively similar to that seen in Table 1. Bond A in Example 2 has an APD of 10% and an RNPD/APD ratio of 3 while the 'lower rated ' bond B has an APD of 20% and an RNPD/APD ratio of only 2.5.

Example 3. Consider Examples 1& 2 and suppose that, in addition, we are given a new bond C whose observed current (date 0) price is 0.618 (or 34/55) and whose future payoffs are 1 in state 1, and 0.6 in states 2 and 3. i.e. bond C defaults in states 2 and 3, and its APD is 30%. As in Example 2 (and footnote 9), its RNPD can be calculated to be 0.8. The resulting RNPD/APD ratio for C is 2.67, and this bond is clearly at odds with the data shown in Table 1.

The new bond here has a higher APD relative to the 'old' bonds of Examples 1& 2 and, yet, has a higher RNPD/APD ratio than one of the old ones. Thus, one ought to ask: how general or pervasive is the phenomenon depicted in Table 1? Alternatively, is there anything perverse about Example 3 which (despite relying only on the absence of arbitrage) appears to capture a case where the RNPD/APD ratio is not necessarily uniformly declining across bonds of different APDs?
Prior work does not address reasons why (and when) the ratio may be decreasing in the bond rating. However, Hull et al. (2005) do point out that differing betas, i.e. risk premia, across ratings may play a role — and before we explore this theme further, we point to a useful interpretation in terms of welfare costs.[11]

[8] Using $q_{d,A} \equiv q(3), q_{d,B} = q(2)$, and $\sum_s q(s) = 1$, these also imply that the risk-neutral probabilities of the individual states 1, 2 and 3 are = 0.2, 0.5 and 0.3 respectively.

[9] Rather than the yield on comparable maturity Treasuries, Hull et al. (2005) use the 7-year swap rate less 10 bps as their riskfree benchmark (which averaged 43 basis points in excess of the Treasury yield in their sample) in line with market norms. This may also correct for non-default risk components in spreads such as those due to tax treatment and illiquidity.

[10] If we did not annualise our probabilities and use their cumulative counterparts over an assumed 7-year maturity,nothing of substance changes in Table 1. The key object of interest, the RNPD/APD ratio, would vary between 1.1 and 16.4 as ratings improve with an average of 6.8.

[11] See Altman (1989), Elton, Gruber, Agrawal, and Mann (2001), Bakshi, Madan, and Zhang (2006), and Chen, Collin-Dufresne, and Goldstein (2009) for more empirical results on the significance of actual default probabilities (or expected losses) relative to bond spreads.

Table 1 Risk-neutral and actual default probabilities.

Rating	APD	RNPD	Actual expected loss	Risk premium	RNPD/APD	RNPD-APD
Aaa	4	67	2	38	16.7	63
Aa	6	78	4	43	13.0	72
A	13	127	8	69	9.8	114
Baa	47	235	28	113	5.0	188
Ba	237	494	142	154	2.1	257
B	722	863	433	85	1.2	141
Caa & Lower	1555	1918	933	218	1.2	364
Average	*369*	*540*	*221*	*103*	*7.0*	*171*

All numbers except for the RNPD/APD ratio are stated in basis points. The APDs are adapted from Hull et al. (2005) using the transformation $p_d = 1{-}e^{-h}$ given its real-world annualised default intensity h (and similarly for the RNPDs). Also shown are the (approximate) actual expected losses $p_{d,j}$ $(1 - R)$ and the (approximate) risk premium $\lambda \approx (q_d - p_d)(1 - R)$.

Risk-neutral probabilities and the cost of insurance

Since bond j pays only a fraction R_j of its face value when default occurs, investors face a possible loss of $1 - R_j$. Consider insurance that makes good this loss (i.e. pays $1 - R_j$) when default occurs. It involves no up-front cost but requires later payment of a known premium θ_j at bond maturity whether or not default occurs. This insurance policy is just a Credit Default Swap (CDS) and the premium θ_j is the 'CDS spread'. An investor who buys 1 unit of risky bond j can, with no additional current out-of-pocket investment, enter into such a CDS or insurance contract and lock in a future riskfree payoff of $1 - \theta_j$ whether the bond defaults or not. Thus, the bond price $B_{0,j}$ and the fair value of the insurance premium must be jointly determined in the market such that strategy's cost $B_{0,j}$ equal $(1 - \theta_j)/(1 + r_f)$. Using (3), with this no-arbitrage restriction now gives $q_{d,j} = \theta_j/(1 - R)$. Thus, the RNPD of a bond is just the cost per unit coverage of insurance against default and the RNPD/APD ratio $q_{d,j}/p_{d,j}$ is, correspondingly, the cost $\theta_j/[p_{d,j}(1 - R)]$ per unit expected loss or coverage. Thus, the data in Table 1 implies that insurance coverage of \$100 million can be obtained with Aaa-rated bonds at a cost of 67 basis points per dollar or \$670,000. However, the same coverage against default of Ba-rated bonds would cost \$4.94 million — only about 7.5 times as much as the cost for the Aaa's despite being 60 times as likely to default. In other words, the ratio of the two probabilities has the direct connotation of capturing costs of insurance and it implies that the striking cross-sectional relative behaviour of the two probabilities is linked to investors' welfare across different types of default events.

A basic identity and conjectures about the two sets of probabilities

From (1) and (3), we may link the risk-neutral probability of default q to the corresponding actual probability p as

$$q_{d,j} \equiv a_j + p_{d,j} b_j = \frac{\lambda_j + p_{d,j}(1 - R_j)(1 + r_f)}{(1 - R_j)(1 + r_f + \lambda_j)} \tag{5}$$

where the coefficients a_j and b_j are determined by the risk premium λ_j, recovery rate R_j and riskfree rate r_f. Note that (5) is a simple affine form with $a_j > 0$ if and only if $\lambda_j > 0$ while b_j is always positive (since positive bond prices ensure $1 + r_f + \lambda_j > 0$). It illustrates *possible* intuitions about the links between the two probabilities. We now enumerate a few such conjectures using the first partial derivatives of $q_{d,j}$ with respect to its arguments and examine its explanatory power for the Hull et al. (2005) data.

First, an RNPD $q_{d,j}$ will exceed its APD counterpart $p_{d,j}$ if and only if the associated risk premium λ_j is positive.[12] As we will see in section 3, this intuition is correct in that it holds unambiguously assuming only the absence of arbitrage. Most RNPDs do in fact typically exceed APDs as evidenced by the data in Table 1 on portfolios or indexes of bonds within each rating group. Whether credit-sensitive claims to negative beta payoffs command lower RNPDs than their APDs has not, to our knowledge, been tested empirically.

Secondly, note that the first partial derivative of $q_{d,j}$ with respect to $p_{d,j}$ is positive for all bonds since $b_j > 0$. This is consistent with the tendency of market observers to attribute changes in the observed RNPD solely to changes in the underlying APD as a cause. But since this partial derivative effect treats other determinants as fixed, it follows that an increase in one probability is not necessarily accompanied by an increase in the other. Only in a relatively benign market environment (viz. 'normal times') where a change in an individual bond's APD is small and its impact on the risk premium is of second-order magnitude, would changes in one be mirrored by changes of the same sign in the other probability with a unit elasticity – rationalising a commonplace intuition.[13]

Third, the partial derivative of $q_{d,j}$ with respect to λ_j is positive and yields the conjecture that increases (decreases) in the required risk premium will increase (respectively, decrease) the RNPD, holding fixed the APD. This intuition is consistent with an increase in investor risk aversion resulting in a lower (higher) price of positive (negative) beta bonds – by leading to a higher market price of risk and increasing the absolute value of the risk

[12] To see this, rewrite (5) as $q_{d,j} - p_{d,j} = a_j(1 - p_{d,j}) = \lambda_j(1 - p_{d,j})/[(1 - R_j)(1 + r_f + \lambda_j)]$.

[13] A test of this hypothesis would involve investigating market reactions to revisions in explicit announcements (by KMV, for instance) of new real-world default probabilities while holding loss given default or recovery fixed.

premium. Furthermore, a resultant lower (higher) bond price that accompanies a change in its risk premium can, under the hypothetical assumption of risk-neutrality, only be explained by an increase in $q_{d,j}$.

Fourth, the partial derivative of $q_{d,j}$ with respect to r_f shows that $q_{d,j}$ is decreasing in the riskfree rate if and only if the risk premium λ_j is positive.[14] This claim is interesting in that, for the typical case where the risk premium is positive, it is consistent with Longstaff and Schwartz (1995)'s theoretical and empirical claims (and those of others) but is arrived at using a different argument and simpler exposition. However, with both this effect and the last one pertaining to changes in the risk premium, caution is advised since changes in the riskfree rate and risk premia are not always independent of each other (which is effectively an assumption implicit in Longstaff and Schwartz) — as when there is an increase in investors' risk aversion.

A last conjecture that can be drawn from the relationship (5) pertains to the effects of a change in the recovery rate R_j. The partial derivative in (5) of $q_{d,j}$ with respect to R_j is positive — suggesting that good news about the prospects for salvaging the assets of an issuer who may default will result in an increase in the market-implied RNPD! This implausible, purported effect highlights the limited use of working with the relationship (5). It ignores the possibility of the risk premium λ_j, and thereby also the RNPD $q_{d,j}$, coming down when R_j is higher because the bond may be less risky in terms of future payouts. Ignoring the endogenous nature of the risk premium λ_j in (5) would lead to other absurdities: note that $q_{d,j}$ will not equal $p_{d,j}$ when the latter is 0 or 1. This last fact clearly points to the non-linearity of the true relationship between the two probabilities.

In summary, while the relationship or identity (5) is useful at a broad level it must be interpreted with care and is likely to be of limited use in cross-sectional or time-series comparisons when there is significant variation in any of the variables involved. A complete assessment would require a treatment of risk aversion and opportunity costs that drive risk premia.

An investor's optimisation

In order to highlight the determinants of risk premia and their effects on risk-neutral probabilities we now consider a typical investor's portfolio optimisation.

Consider the framework of the second section over dates 0 and T with multiple risky assets $j = 1,\ldots,N$ of which at least one is a default-sensitive bond; there is also a default-free bond with riskless interest rate r_f. Bond j's payoff $B_{T,j}(s)$ is either 1 in states $s \in n_j$ that form the no-default event n_j or $R_j < 1$ in states comprising the default event d_j.[15] The payoffs and current price $B_{0,j}$ give the risky returns $\{r_j(s)\}$: $B_{0,j}(1 + r_j(s)) \equiv B_{T,j}(s)$.

[14] This partial here is $\lambda_j[p_{d,j}(1 - R_j) - 1]/(1 - R_j)(1 + r_f + \lambda_j)^2$. The Longstaff-Schwartz argument is based on the usual comparative static exercise that holds current firm value fixed and concludes that a higher r_f raises the risk-neutral drift of future asset value and thus lowers the RNPD.

[15] For simplicity, we use the index j to refer to both a typical bond and an arbitrary asset.

An investor's initial wealth, set at 1 for simplicity, is allocated across the risky assets according to chosen portfolio weights $x = \{x_j\}$ with a position in the riskfree asset of $1 - \sum_j x_j$. The resulting wealth at date T varies across states and is given by

$$W(s;x) \equiv \sum_j x_j(1 + r_j(s)) + \left(1 - \sum_j x_j\right)(1 + r_f).$$

His attitudes towards risk are described by a utility function $u(.)$ over wealth that is strictly increasing, strictly concave, and differentiable. That is the investor prefers more wealth to less and is risk-averse. The investor chooses a portfolio x to maximise his expected utility of wealth defined, relative to the actual probabilities $\{p(s)\}$, as

$$E^p[u(W(x))] \equiv \sum_s p(s)u(W(s;x)).$$

Let x^* be his optimal portfolio; the necessary (and sufficient) condition for optimality is

$$E^p\left[u'(W(x^*))(r_j - r_f)\right] = 0 \tag{6}$$

where $u'(W(s;x^*))$ is the marginal utility derived from an extra unit of wealth in state s given the chosen optimal portfolio x^*.[16] Condition (6) can be used to solve for optimal portfolios. Alternatively, given an optimal portfolio or associated wealth across states, (6) can be used to determine the price or RNPD of a risky bond in terms of risk aversion and opportunity costs arising from available returns from other assets.

A fundamental relationship

One can rewrite (6) for bond j, grouping states into default or no-default events d_j and n_j:

$$B_{j0} = \frac{\sum_{s \in n_j} p(s)u'(W(s;x^*)) \cdot 1 + \sum_{s \in d_j} p(s)u'(W(s;x^*))R_j}{E^p[u'(W(x^*))](1 + r_f)}. \tag{7}$$

Note that $\sum_{s \in d_j} p(s) \equiv p_{d,j}$ is the APD of bond j. A comparison of (7) with the usual risk-neutral valuation expression (1) then yields a fundamental relationship between (and defines) the risk-neutral probability measure $Q \equiv \{q(s)\}$ in terms of the actual probability measure $P \equiv \{p(s)\}$. An *individual state s's* risk-neutral probability $q(s)$ scales its actual probability $p(s)$ according to:

$$q(s) = p(s)u'(W(s;x^*))/E^p[u'(W(x^*))]. \tag{8}$$

In words, a risk-neutral probability for any state s is a product of both how likely the state is in terms of its actual probability and a scaling factor $\eta(s) \equiv u'(W(s;x^*))/E^p[u'(W(x^*))]$ which is just the marginal utility in that state relative to its average value. The latter factor takes different values across states and is accordingly termed a stochastic discount factor or pricing kernel in modern asset pricing theory (and the Radon—Nikodym derivative of the two measures P and Q in

[16] We have assumed a frictionless setting here. Also, all required mathematical niceties such as the existence of an optimum have been implicitly assumed to hold.

probability theory). Its intuitive feature that 'bad' ('good') states are characterised by low (high) wealth and have $\eta\ (s) > 1$ (respectively, <1) and the implications for default probabilities will be explained shortly.

Adding the risk-neutral probabilities over relevant states then determines the RNPD *for an individual bond j*: $q_{d,j} \equiv \sum_{s \in d_j} q(s)$. Now, the relationship between the two PDs for bond j may be written, from (7) and (8), as

$$q_{d,j} = p_{d,j} E^p\left[u'(W(x^*))|d_j\right] / E^p[u'(W(x^*))]. \tag{9}$$

To interpret and gain insight into (9), we proceed by examining several special cases.[17]

RNPD/APD ratios and systematic risks in the cross-section of bonds

Consider a default-prone bond j, but drop all bond j-specific indexes below (unless otherwise mentioned), and take its default event d to correspond to a single state for simplicity. The investor has chosen his optimal portfolio. The associated wealth in d is denoted $W(d)$ with marginal utility $u'(W(d))$. Thus, the numerator of (9), is then just $p_d u'(W(d))$. The investor's portfolio's terminal value and wealth $W(s)$ will in general differ across the multiple states $s \neq d$ in which bond j does not default with correspondingly different marginal utilities $u'(W(s))$. The expected marginal utility conditional on no-default $E^p[u'(W_T(x^*))|n]$ is some quantity denoted ψ and given by $(1-p_d)\psi = \sum_{s \in n} p(s)\, u'(W(s))$. Hence the denominator of (9) becomes $p_d u'(W(d)) + (1-p_d)\psi$.[18] Thus, the bond's RNPD from (9) is

$$q_d = \frac{p_d u'(W(d))}{p_d u'(W(d)) + (1-p_d)\psi}. \tag{10}$$

With this relationship — a ratio of two linear functions - a number of results now follow which are contained in the rest of the third section. It is useful to compare them with those stemming from the naïve relationship of the second section.

First, q_d is positive and less than 1 if and only if the same is true of p_d because marginal utilities are positive. Also, q_d is 0 or 1 when p_d is 0 or 1, respectively.

Next, the RNPD is the APD scaled up or down by a factor that compares $u'(W(d))$ with $p_d u'(W(d)) + (1-p_d)\psi$ thereby capturing attributes of the particular bond and consequences of its default. Both terms also depend on the investor's risk aversion, measured through $u'(.)$, and his overall portfolio's performance across possible macroeconomic states as measured by actual probability beliefs and the associated asset returns implicit in $W\ (d)$ and ψ. Specifically, the RNPD-to-APD ratio compares the expected reduction in utility, $u'(W(d))$, per \$1 loss from the bond's default to the expected decrease in utility $p_d u'(W(d)) + (1-p_d)\psi$ arising from a sure loss of \$1 from future wealth. An explanation of this comparison now follows.

Since the investor is risk-averse, his marginal utility is higher the lower his wealth. Suppose the investor is relatively poor in the state where the bond defaults in the sense that $u'(W(d)) > \psi$, i.e. $W\ (d)$ is sufficiently low relative to wealth $W\ (s)$ in non-default states $s \neq d$. Then the value or welfare loss suffered per \$1 loss due to default exceeds the average value $p_d u'(W(d)) + (1-p_d)\psi$, and the larger this excess the more default will hurt him. Thus, ex ante, he penalises such a bond more than what consideration of solely its odds of default would suggest. Consequently, its lower price is reflected in an RNPD q_d that will exceed its APD p_d.

It is also useful to consider the risk premium of the bond. Combining the basic identity (5) with (10), the risk premium to actual expected loss of the bond is

$$\frac{\lambda}{p_d(1-R)} = \frac{(1-p_d)(1+r_f)[u'(W(d)) - \psi]}{p_d u'(W(d)) + (1-p_d)\psi} \tag{11}$$

Thus, the risk premium is positive or negative according to whether the bond's default is in a state where $u'(W(d))$ exceeds or falls short of ψ. Accordingly, we may say that such a bond has a positive 'beta': it pays when his wealth is high and defaults when his wealth is low.[19]

Nothing precludes an individual bond from having an RNPD less than its APD. Suppose, in contrast to our earlier assumption, the bond's default event occurs when the investor is relatively rich (where his overall portfolio does well) in the sense that $u'(W(d)) < \psi$, i.e. $W\ (d)$ is sufficiently high relative to wealth $W\ (s)$in non-default states $s \neq d$. This would be the case for the debt of a negative beta business which is more likely to not pay in good macroeconomic times than bad ones. Using the same logic as earlier, we may conclude that for this bond $q_d < p_d$. Similarly, Example 3's bond C is not perverse relative to B since it also defaults in state 3 whose q-to-p ratio is high, i.e. when an extra dollar in wealth has larger marginal utility value than average.

A bond whose default is independent of an investor's portfolio can be treated similarly. A necessary condition for this is that the bond's prospects are independent of the returns of other assets held and the investor's exposure to it is a small fraction of his portfolio, i.e. he diversifies its risk with positions in other similar assets. Then his wealth when the bond defaults is not expected to be any different from when it does not default. Under such circumstances, the default event is uncorrelated with his marginal utility of wealth and the bond has a zero beta. Then, from (10), $u'(W(d)) = \psi$ and hence $q_d = p_d$. The classic example of this is insurance that pays contingent on a risk that can be eliminated when aggregated over a large number of policy-holders. Risks of this

[17] Wirh the stochastic discount factor here given by $m(s) = u'(W(s;x^*))/E^p[u(W(x^*))]$, Eq. (8) becomes $q(s)/p(s) = m(s)$. Eq. (9) is a conditional version of (8), given the event that bond j defaults.

[18] Using the definition of a conditional expectation, $E^p[u(W_T(x^*))] = p_d E[u'(W_T(x^*))|d] + (1-p_d)E[u'(W_T(x^*))|n]$ where d and n are the default and no-default events of the bond.

[19] To see this formally, write the conditional expectation $E^p[u'(W(x^*))|d_j]$ as $E^p[u'(W(x^*))]$ less the covariance with $-u'(W(x^*))$ of an indicator random variable defined relative to the default event. These are not the usual CAPM betas but generalised versions representing the covariance of the bond's payoff with the investor's portfolio expressed in marginal utility terms.

sort, e.g. theft, fire, etc, are priced by the market exactly in line with actuarial odds. Obviously, most assets' returns carry exposure to macroeconomic or systematic factors and hence their risks are not completely diversifiable.

Third, differentiating the RNPD in (10) with respect to the APD p_d one can verify that the higher the APD the larger is the RNPD (for both positive and negative beta bonds). Fourth, differentiation a second time, shows that q_d is concave (convex) in p_d and this implies that bonds' RNPD/APD ratios gradually decrease (increase) to 1 as the actual probability of default increases provided $q_d > p_d$ (respectively, $q_d < p_d$). Equivalently, as can be seen by differentiating (11), the risk premium to actual expected loss ratio comes down as the APD increases if and only if the bond has a positive beta. This generalises the Berg (2010) result — which considered only securities with positive betas — from a Merton (1974) framework to a very basic setting and thus demonstrates its robustness. It simultaneously shows that the role of the bond's risk premium or beta is crucial.

In summary, Eq. (9) says that the RNPD $q_{d,j}$ of an individual bond j is a scaled up or down version of its APD $p_{d,j}$. The relevant scaling factor here is just a conditional version, $E^p[u'(W_T(x^*))|d_j]/E^p[u'(W_T(x^*))]$, specific to bond j of the earlier scaling factor or pricing density η of (8). The conditional expectation $E^p[u'(W_T(x^*))|d_j] \equiv [\sum_{s \in d_j} p(s)u'(W(s;x^*))]/p_{d,j}$ is the investor's expected marginal utility conditional on default of bond j and is a measure of the bond's systematic risk. Thus, the behaviour of the ratio $q_{d,j}/p_{d,j}$ of the RNPD to the APD varies in the cross-section of bonds $j = 1,\ldots,N$ according to how any two bonds j and k differ in the relative magnitudes of $E^p[u'(W_T(x^*))|d_j]$and $E^p[u'(W_T(x^*))|d_k]$. Some bonds' default events will have a high covariance with the investor's marginal utility across states and high betas while others will have low betas. Accordingly, the ratio of the PDs for the former will exceed that for the latter.

This explains the data in Table 1 which pertains to bond indexes. i.e. they represent the average bond in a given ratings class. The fact that the ratio exceeds 1 can be explained by arguing that the average bond in any ratings class most plausibly has a positive 'beta'. Our explanation also implies however that the RNPD can well be less than an APD for some individual bonds since systematic risks can and do differ across individual securities. If the creditworthiness of the issuer is negatively correlated with the overall macroeconomy, and thus with investor's portfolios, the default of such a bond hurts less since its default event is likely to occur when the investor's portfolio is, for other reasons, doing well.

A similar explanation underlies the behaviour of structured products created as claims on pools of individual bonds — only the quantitative effect would be even stronger. Coval et al. (2008) show that high values of the RNPD/APD ratio extend beyond indexes of corporate bonds to market prices of structured products such as tranches of CDOs written on similar indexes. Furthermore, consistent with the systematic risk explanation, the non-linear pattern discussed in this paper is stronger in their paper for CDOs: tranches with APDs comparable to AAA-rated bond commanded RNPD/APD ratios of between 18 and 60 as opposed to Table 1's number of 17 during the years immediately before and during the crisis. This huge difference can be explained by noting that the numbers in Table 1 refer to a pro-rata share in a pool (i.e. index) whereas CDOs allocate losses according to a priority rule and thus their senior tranches are even more sensitive to systematic risk than the underlying pool (or bond index). Taking note of the qualitative similarity between bonds (our paper) and structured products (Coval et al) in the non-linear patterns in their RNPD/APD ratios supports the view that CDO investors did not pay attention to low APDs alone but were (at least partially) correct in seeing senior tranches as carrying large systematic risk exposure despite their low APDs. However, Coval et al argue tranches were nevertheless still overvalued in the CDO market. Conditional on the assumption that the relevant systematic risk can be proxied by covariances relative to the S&P 500, they show the ratios seen in the market were too small relative to the prices of hypothetical put options securities created with similar payoffs similar to CDO tranches.

The impact of news

The analysis above can be readily used to explain how an RNPD should change when there is a news event that is particular to a single bond. i.e. it allows one to evaluate the frequent criticism made by ratings agencies of market-implied probabilities, viz. that they are excessively volatile.

We saw in the section '*A basic identity and conjectures about the two sets of probabilities*', working with the basic identity (5), that an increase in APD leads to a higher RNPD for all bonds regardless of their beta, i.e. whichever of the two probabilities exceeded the other. But we also pointed out that (5) was only a rudimentary relationship and the claimed effect used an assumption that an increase in APD would have no effect on the bond's risk premium. With the benefit of the richer relationship (9) that endogeneises effects on the risk premium, we now see that the claim made in the second section is unambiguously true — but with an interesting twist. Consider the investor and bond of the sub-section 'A fundamental relationship' when actual probabilities are $\{p\ (s)\}$ prior to a news event. Suppose the news event is a 'bad' one that increases the actual probability p_d of the state in which bond j defaults to $\hat{p}_d$.[20] Further, suppose the news event is an 'own' one: it does not affect the relative probabilities of other states but simply re-scales them to their new sum which equals the lower probability of no-default $1 - \hat{p}_d$ of bond j. Such news does not change the marginal utility $u'(W(d))$ in the default state. Hence, it should be clear on inspection that the expected marginal utility conditional on no-default also retains its prior value of ψ.[21] It follows that the revised

[20] The treatment here, implicitly, examines a dynamic version of the static framework of the earlier sections. It underscores the point that the essential results and intuition are unchanged. We do not formally adopt a full-blown dynamic treatment in the interests of simplicity.

[21] Use $\hat{p}(s) = p(s)[(1-\hat{p}_d)/(1-p_d)]$ to verify that $E^p[u'(W_T(x^*))|n] = \psi$. It follows that $E^p[u'(W_T(x^*))]$ becomes $\hat{p}_d u'(W(d)) + (1-\hat{p}_d)\psi$. Analysis of the case where the bad news also lowers the recovery rate R can be done similarly.

RNPD $\hat{q}_d$ of the bond after the news event has exactly the same form as in (10) above with $\hat{p}_d$ replacing p_d. Thus, such news that impacts only the bond's PD - and does not impact its recovery rate or the APDs of other bonds or returns of other assets — can be evaluated as per our earlier discussion in the previous sub-section (*RNPD/APD ratios and systematic risks in the cross-section of bonds*).

It follows from the first partial derivative of q_d with respect to p_d discussed earlier that an increase in APD from p_d to $\hat{p}_d$ results in an increase in RNPD if and only if the bond has a positive beta. The numbers in Table 1 may be used as illustration. Consider the typical bond which has RNPD/APD > 1 (i.e. a positive beta bond). A given increase in the APD of 15 bps, say, from 1.5% to 1.65%, is accompanied by a larger increase in the RNPD of 75 bps, say, from 15% to 15.75%. (Clearly, from Table 1, at a low enough initial level of APD, the RNPD/APD ratio is 5). So in this sense the ratings agencies criticism has merit — the market-implied counterpart is 'excessively' volatile at low APD levels. However, reaction by the market gets progressively muted for every successive increase in the APD until a point where further worsening of actual default prospects are not matched to an equal degree by the market.

A further point of interest comes from noting that investment grade bonds have, on average, large values of the ratio relative to speculative grade bonds in Table 1. Thus, the sensitivity of RNPDs (i.e. spreads) to news is greatest for the 'safest' bonds and lower rated bonds' RNPDs should be less volatile than their APDs. This may be understood from the second partial derivative of q_d with respect to p_d which is negative under the same condition that beta > 0 or the RNPD/APD ratio > 1. Thus, the impact on q_d of bad news about p_d is strictly increasing and in a strictly concave manner for such bonds. In words, the marginal impact of an increase in p_d on q_d is greatest when the APD is lowest (and the bond beta is positive). We have shown that this may be understood along the lines that bad news about a bond is more serious the safer the bond was thought to have been.

Further discussion

What's special about credit?

A pertinent question is what, if anything, is special about risky bonds or credit derivatives vis-à-vis distinctions between the actual and risk-neutral probabilities? After all, risk premia exist everywhere and consequently the two probabilities must in general differ. Thus, if distinctions between risk-neutral and actual probabilities can be ignored in other markets perhaps they can be glossed over in credit markets too.

Credit securities present an interesting case because of the predominantly binary or discrete characteristics attached to key underlying or trigger events. For instance, a risky bond or loan will either default or not do so, and a debt covenant will either be breached or satisfied. Furthermore, regulatory or prudential restrictions on a money manager's portfolio choices often hinge on events with discrete outcomes like ratings changes, and are thus relevant to all credit investors. Bank capital regulations are often prescribed with reference to actual probabilities. A firm's cost of capital is influenced by its rating. And so on. When pre-specified discrete events are material to an investor or a security, it is natural to attach importance to actual probabilities of these events. Furthermore, it is typically assumed in practice that a bond's payoff in the default event is a known (as a first order approximation) fraction, say 40%, of its full or promised payment. By assuming away the uncertainty associated with the actual payment that an investor may recover from a defaulted bond, attention then devolves on the easier task of assessing the probability of default, per se, rather than the relevant moment (probability times the payoff or the loss). To this extent, credit securities differ from other assets such as stocks or options where one can't dispense with the need to consider moments. These characteristics of credit securities also explain why there is widespread production of information tied to actual default probabilities such as ratings put out by agencies such as Standard & Poor's and Moody's.

Why risk-neutrality?

But why risk-neutral implied probabilities? Why not seek to construct market-implied probabilities taking investor risk aversion or risk premia into account? Is the latter task even feasible?

Our earlier simple formulation in Eq. (1) illustrates that an observed bond price (or yield) can be used to estimate a market-implied APD provided one takes a 'view' on the bond's risk premium λ. Consistent with this approach, Moody's KMV, a business founded on the Merton conceptualisation of debt, sells APDs that make use of market prices. Using a firm's stock prices they derive RNPDs for corporate debt as per the Merton logic. In the next crucial step, they use a proprietary database of past history on the RNPD-APD relationship to map an RNPD to its corresponding APD. It is difficult to see how this is not, at least in part, equivalent to making an assumption about the expected future return of the firm's assets based on past data. Efforts to replicate KMV's approach make direct assumptions of this sort which are effectively assumptions about the firm's risk premium from which, in the Merton framework, that of debt follows.[22]

Our earlier remarks point to the role of risk premia and how they are driven by investors' risk aversion, the likely returns on other securities available, and the current values of their portfolios. Other features of their circumstances such as any future liabilities, ability to short sell, regulation, etc also matter. None of these characteristics is perfectly observable and hence must be estimated. Furthermore, these may change with time and the macroeconomic environment. While the theory of asset pricing

[22] After a successful history as an independent business, competing with ratings agencies, KMV was acquired by Moody's and is now known as Moody's KMV. To all appearances, Moody's KMV makes heavy use of recent past returns of a firm's assets, as implied by its equity performance in formulating this return on assets.

provides guidance in order to conduct inferences along these lines, it should be clear that this is a delicate task.

This explains one reason why market-implied risk-neutral probabilities are popular — they are easy to construct, and with proper awareness, can be used in a discriminate manner. A further reason is simply that most credit risk valuation models are based on the risk-neutral methodology first developed in the context of option pricing. Their objective is that of determining the price of a candidate security in terms of given market prices of other assets (and assumed characteristics of the future stochastic evolution of the latters' prices). Importantly, these models make no direct use of, and thus have no transparent role for, actual probabilities.[23] Note that this does not do away with the need for actual probability estimates. 'Active' real-world investment decisions involve views about future likely scenarios and their effects on returns. For instance, an investor taking a short-term position today by buying a distant maturity corporate bond will almost certainly find it useful to assess the position's expected return (and volatility) over the near term — based on actual probabilities.

Policy relevance

As noted in the Introduction, the use of market prices to draw inferences has been used at all times, arguably, in the history of financial markets. All such efforts are (at least implicitly) based on specific models such a Fisherian hypothesis with interest rates, or the Gordon dividend discount model with stocks, or the Black and Scholes (1973) model with implied stock volatilities. As argued earlier, it is impossible to construct any 'model-free' expectations from market prices. Going by the voluminous commentary based on such signals, however, many appear to be insufficiently appreciative of the foundations of such approaches. Thus, it is very important to be circumspect about arguments for market-implied ratings such as those from Flannery et al. (2010).

Empirical implications and further work

One result of ours implies that systematic risk is less important for high-yield bonds relative to investment grade bonds after accounting for the larger APDs of the former. This is consistent with Cornell and Green (1991) and Fama and French (1993) estimates of betas of bonds across different ratings. Their results suggest that the beta of a Ba bond is about two-to-five times as large as that of an Aaa despite its actual default probability being 60 times larger. We also argued that bonds with low APDs should, on average, be more sensitive to macro news. Evidence consistent with this view is found in CDS spreads for high-rated versus low-rated tranches created from CDOs during the recent crisis. Scheicher (2010) notes that CDS spreads for the senior-most tranche went up by a factor of 48 from June 2007 to September 2008 (soon after the Lehman Brothers default). In contrast, over the same period, spreads of the junior-most tranche increased by a relatively modest factor of about 3. Further work can seek to directly empirically examine implications of both our results using appropriate direct tests on corporate bonds.

Concluding remarks

We have reviewed the meaning of risk-neutral probabilities in general with a particular focus on credit markets and their relationship with actual probabilities. Our analysis is intended to facilitate a more nuanced understanding of the relationship between the two probabilities. RNPDs can differ across issuers not just because their underlying APDs may differ but also with changing (absolute) risk aversion over time and with varying macroeconomic states. This has implications for the cross-sectional behaviour of market-implied probabilities. For instance, the market's reaction to bad news about default prospects should be stronger for a bond the lower that its APD is. Bonds whose defaults occur in poor macroeconomic circumstances hurt investors more. One bond's default would be expected to hurt less than that of another if the former default event is expected to coincide with states where the investor's portfolio is, for other reasons, doing well. The main implications of our story are consistent with certain characteristics of market data that appear to have puzzled several observers. Other properties were set out using simple economic analysis.

It is important to recognise that our results are consistent, in a qualitative sense, with a modelling approach that is more quantitative and of the sort typically used in derivatives valuation — whether of the structural or reduced form types, and whether set in continuous or discrete time, and also with regard to other refinements such as incomplete information about an issuer's liabilities. Essentially, the crux of asset pricing theory's central economic implication applies even for individual credits — systematic risk matters and will impact the ratio of a bond's RNPD to its APD.

We have described credit risk pricing based on an assumption of at least some rational investors in a (state-preference) framework that is common to neoclassical financial economics even if under-appreciated amongst practitioners.[24] Two other ways of expressing the lessons herein are as follows. First, rare or so-called tail risks (e.g. default of an Aaa bond) *are, in fact*, priced at much more than a simple consideration of their historical odds of occurrence may suggest relative to the cost of less infrequent risks (e.g. default of a Ba bond). This message is significant given a barrage of ill-informed views, in our

[23] Relationships between risk-neutral and actual probabilities are, effectively, subsumed in such models to the extent that they treat actual expected returns. However, starting with Black and Scholes (1973), it has been well-known that distinctions between the two probabilities (and the 'drift') do not matter to option values (in terms of prices of other assets) at a given time, if the option can be perfectly hedged or replicated.

[24] Another assumption we have made, that is also common to this approach, is that of no frictions. Relaxing this would lead to clientele effects and segmentation in markets both of which are, to one degree or another, familiar in the credit arena.

opinion, since the recent credit crisis about the deficiencies of pricing models. Another implication relates to how someone who does not understand (or believe in) the systematic risk rationale may approach investing. If he sees Aaa bonds as being priced efficiently, then Ba bonds may appear to be priced too high and such a 'conservative' investor may shy away from long positions in 'junk' debt. Conversely, to an investor who views Ba bonds as efficiently valued, Aaa bonds may appear to be priced too low: such an investor accustomed to high yields from lower rated debt may seek similar expected returns through leveraged purchases of investment grade bonds without understanding the true risks. Both these implications are distressingly familiar given the recent credit crisis.

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